

Video Denoising Based on Riemannian Manifold Similarity and Rank-One Projection

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Abstract

In this paper, we combine two powerful tools to handle the video denoising problem: one is an effective video denoising method based on Riemannian Manifold Similarity, and the other is a Rank-One Projection matrix completion based on video denoising method. Similarly, in our algorithm, a noisy video is processed in block-wise manner for each processed block. we form a 3D data array that we call “group” by stacking together blocks which is found similar to the currently processed one. “Collaborative filtering” exploits the correlation between grouped blocks and the corresponding highly sparse representation of the true signal in the transform domain. By employ Rank-One Projection matrix completion method in our framework, our technique is also robust to different types of noise. Experiments demonstrate that our techniques produce state-of-the-art results for video denoising applications.

Keywords

Video Denoising, Rank-One Projection Matrix Completion, Riemannian Manifold Similarity, Block Matching

Introduction

With today’s advances in sensor design, the image/video is relatively clean for high-end digital cameras at low sensitivities, but it remains noisy for low cost cameras at high sensitivities, e.g., low light condition, high ISO setting and high speed rate. The problem of removing image noise is still of acute and in fact growing importance with the prevalence of webcams and mobile phone cameras. A recent denoising strategy, the non-local spatial estimation [2], has also been adapted to video denoising [3]. In this approach, similarity between 2D patches is used to determine the weights in a weighted averaging between the central pixels of these patches. For image denoising, the similarity is measured for all patches in a 2D local neighborhood centered at the currently processed coordinate. For video denoising, a 3D such neighborhood is used. The effectiveness of this method depends on the presence of many similar true-signal blocks[4].Based on the same assumption as the one used in the non-local estimation, i.e. that there exist mutually similar blocks in natural images, in [5] the authors proposed an image denoising method. There, for each processed block, we perform two special procedures: grouping and collaborative filtering. Grouping finds mutually similar 2D blocks and then stacks them together in a 3D array that we call group. The benefit of grouping highly similar signal fragments together is the increased correlation of the true signal in the formed 3D array. Collaborative filtering takes advantage of this increased correlation to effectively suppress the noise and produces estimates of each of the grouped blocks. They showed [5] that this approach is very effective for image denoising. In this paper, we apply the concepts of grouping and collaborative filtering to video denoising. Grouping is performed by a specially developed predictive-search block matching technique that significantly reduces the computational cost of the search for similar blocks. We employ a two-step video-denoising algorithm proposed in [4] where the predictive search block-matching is combined with collaborative hard thresholding in the first step and with collaborative Wiener filtering in the second step. In order to enhance the robustness of the algorithm in processing denoising problems with multiple sources of noises, our algorithm is derived with minimal assumptions on the statistical properties of image noise. The basic idea is to convert the problem of removing noise from the stack of matched patches to a low rank matrix completion problem, which can be efficiently solved by minimizing the nuclear norm of the matrix with linear constraints.[1]

Related Work

There has been an abundant research literature on image denoising methods. In this section, we will only discuss the most related denoising techniques. There has been abundant research literature on image denoising methods. In this section, we will only discuss the most related denoising techniques [4,6,7]. Although differing from details, these method are built on the same methodology which essentially groups the similar patches together followed by a collaboratively filtering. Take the well-known BM3D [5] as a sample. In BM3D, similar image blocks are stacked in a 3D array based on the L2 norm distance function between different patches. Then a shrinkage in 3D transform domain such as wavelet shrinkage or Wiener filter is applied on the 3D block stack. The denoised image is then synthesized from denoised patches after inversing 3D transform. The result can be further improved by iteratively doing grouping and collaboratively filtering. Video denoising is different from single image denoising as video sequences usually have very high temporal redundancy which should be effectively used for better performance (e.g., [8-10]). The basic idea of patch-based image denoising can also be applied on the video by matching similar patches both within the image and over multiple images. The concept of BM3D is generalized to video denoising in [4] by using a predictive search block-matching over time and combined with collaborative Wiener filtering on patch stacks. In [11], a more robust patch matching is proposed by using the depth as a constraint in the matching process and the patch stack is denoising by both PCA (principle component analysis) and tensor analysis. The idea of sparse coding in a patch dictionary has also been applied on video denoising, where the denoised image patches are found by seeking for the sparsest solution in a patch dictionary. Among these patch-based video denoising techniques, most assume data noise is only additive i.i.d. Gaussian noise (e.g., [4]). The image noise mixed with both Gaussian noise and Poisson shot noise are considered in [14].

Our Approach

Image comparison is a topic that has received a lot of attention in the image processing and computer vision communities since it is a main ingredient in many applications, such as object recognition, stereo vision, image interpolation, image denoising, and exemplar-based image inpainting, among others. A common way to define a nonlocal similarity measure between two images is to compare the patches (local neighborhoods) around each pair of points formed by taking one point from each image. We consider a general setting in which images are defined on Riemannian manifolds. Such manifolds arise, for instance, for images defined on \mathbb{R}^N , endowed with a suitable metric depending on the image. In [3], it was shown that multiscale analyses of similarities between images on Riemannian manifolds, satisfying a certain set of axioms, are (viscosity) solutions of a family of degenerate PDEs. Our goal in this paper is to study one particular instance of the set of models derived in [3], namely a linear model to compare patches defined on two images in \mathbb{R}^N endowed with some metric. Except for its genericity, this linear model is selected by its computational feasibility since the solution of the PDE can be approximated via the convolution with a short-time space-varying kernel, leading to an algorithm that has the complexity of the usual Euclidean patch comparison. Let us review the fundamentals of the approach in [3]. Given two images u, v defined in their respective image domains (assume \mathbb{R}^2 for simplicity), we want to compare their neighborhoods at the points $x, y \in \mathbb{R}^2$, respectively. The simplest way to compare them would be to compare the two neighborhoods of x, y using the Euclidean distance. That is, let us define

$$D(t, x, y) = \int_{\mathbb{R}^2} \mathcal{G}_t(h) (u(x+h) - v(y+h))^2 dh, \quad (1)$$

where \mathcal{G}_t is a given windowing function that we assume to be Gaussian of variance t . This formula gives an explicit comparison and assumes that the image domain is the Euclidean plane. It generalizes approaches to patch comparison applied, for example, in [8].

In this part, we employ the strategy utilized in [4] to solve our problems. In order to efficiently capture blocks that are part of objects which move across subsequent frames, we propose the use of linear multiscale analysis of similarities between images on Riemannian manifold which finds similar (matching) blocks by searching in a data adaptive spatio-temporal subdomain of the video sequence. For a given reference block located at $x = (x_1, x_2, t_0)$,

when using a temporal window of $2N_{FR} + 1$ frames.

For each group formed in the former stage, a patch matrix is constructed by concatenating every patch in the group into a long vector and stacking all the vectors. With the purpose of restoring the patch matrix, we adopt a similar method discussed in [1]. For each patch, similar patches are found in both spatial and temporal domain by using the patch matching algorithm described in the previous section to form the matrix $P_{j,k}$. The set of missing elements of $P_{j,k}$ have two subsets: the first part covers those pixels corrupted by impulsive noise using the adaptive median filter based impulsive noise detector ([9]). The second part includes the pixels whose value differs from the mean of the corresponding row vector by the amount larger than a pre-defined threshold. Then Ω is formed by including the index of all remained pixels. As described in [1], $Q_{j,k}$ is recovered from its incomplete observation $P_{j,k}|_{\Omega}$ by solving the following minimization problem:

$$\begin{aligned} \min_Q \|Q\|_* \\ \text{s.t. } \|Q|_{\Omega} - P|_{\Omega}\|_F^2 \leq \#(\Omega)\hat{\sigma}^2, \end{aligned} \quad (2)$$

where $\hat{\sigma}$ is the estimate of standard deviation of noise, which is obtained by calculating the average of the variances of all elements $\in \Omega$ on each row.

we introduce a "Rank-One Projection" (ROP) model for low-rank matrix recovery and a constrained nuclear norm minimization method for this model. Under the ROP model, we observe

$$y_i = (\omega^i)^T A \theta^i + z_i, \quad i = 1, \dots, n, \quad (3)$$

where ω^i and θ^i are random vectors with entries independently drawn from some distribution P , and z_i are random errors. In terms of the linear map $X: \mathbb{R}^{p_1 \times p_2} \rightarrow \mathbb{R}^n$ in (1.1), it can be defined as

$$[X(A)]_i = (\omega^i)^T A \theta^i, \quad i = 1, \dots, n, \quad (4)$$

Since the measurement matrices $X_i = \omega^i A (\theta^i)^T$ are of rank-one, we call the model (3) a "Rank-One Projection" (ROP) model. It is easy to see that the storage for the measurement vectors in the ROP model (3) is $O(n(p_1 + p_2))$ bytes which is significantly smaller than $O(np_1 p_2)$ bytes required for the Gaussian ensemble. We first establish a sufficient identifiability condition in Section 2 by considering the problem of exact recovery of low-rank matrices in the noiseless case. It is shown that, with high probability, ROP with $n \gg r(p_1 + p_2)$ random projections is sufficient to ensure exact recovery of all rank- r matrices through the constrained nuclear norm minimization. The required number of measurements $O(r(p_1 + p_2))$ is rate optimal for any linear measurement model since a rank- r matrix $A \in \mathbb{R}^{p_1 \times p_2}$ has the degree of freedom $r(p_1 + p_2 - r)$. The Gaussian noise case is of particular interest in statistics. We propose a new constrained nuclear norm minimization estimator and investigate its theoretical and numerical properties in the Gaussian noise case. Both upper and lower bounds for the estimation accuracy under the Frobenius norm loss are obtained. The estimator is shown to be rate-optimal when the number of rank-one projections satisfies either $n \gg (p_1 + p_2) \log(p_1 + p_2)$ or $n \sim r(p_1 + p_2)$. The lower bound also shows that if the number of measurements $n < r \max(p_1, p_2)$, then no estimator can recover rank- r matrices consistently. The general case where the matrix A is only approximately low-rank is also considered. The results show that the proposed estimator is adaptive to the rank and robust against small perturbations. Extensions to the sub-Gaussian design and sub-Gaussian noise distribution are also considered.

Experiments and Conclusion

We applied our proposed denoising method on several videos with different mixed noise levels. The results are compared with that of one existing video denoising method VBM3D [4]. The same algorithm parameters were set as recommended in [4]. It is obvious that the proposed method shows superior preservation of fine image details and at the same time it introduces significantly less artifacts. In this work, we combine two powerful tools to handle the video denoising problem: one is an effective video denoising method based on Riemannian Manifold Similarity, and the other is a Rank-One Projection matrix completion based on video denoising method. Similarly, in our

algorithm, a noisy video is processed in block-wise manner and for each processed block. This work is supported in part by 973 projects (2012CB725305) and the National Key Technology R&D Program projects (2012BAH70F02, 2013BAH27F03).

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